

Dynamical symmetry breaking in SYM theories as a non-semiclassical effect^{*}

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Abstract. We study supersymmetry breaking effects in $N = 1$ SYM from the point of view of quantum effective actions. Restrictions on the geometry of the effective potential from superspace are known to be problematic in quantum effective actions, where explicit supersymmetry breaking can and must be studied. On the other hand the true ground state can be determined from this effective action, only. We study the problem whether some parts of superspace geometry are still relevant for the effective potential and discuss whether the ground states found this way justify a low energy approximation based on this geometry. The answer to both questions is negative. Essentially non-semiclassical effects change the behavior of the auxiliary fields completely and lead to the demand of a new interpretation of superspace geometry. These non-semiclassical effects can break supersymmetry.

1 Introduction

The question whether supersymmetry is spontaneously broken or not is of fundamental importance. Many results concerning this problem have been derived in the literature. We know that perturbative corrections do not break supersymmetry. What happens non-perturbatively is not yet clear since there is no mathematical tool available to describe this regime. Our knowledge of the behavior of non-Abelian gauge theories is restricted to perturbative results, semiclassical analysis and simulations on the lattice. But the ground state of any non-Abelian gauge theory that is not broken down completely (up to $U(1)$ factors) is characterized by non-perturbative effects. Supersymmetry does not help us in this situation. On the contrary: many supersymmetric models (e.g. $N = 2$ SYM) are completely unacceptable in the perturbative region, as the perturbative β function develops a Landau pole. Exploring the non-perturbative region is not at all simpler than in ordinary QCD: measurements from the lattice are not yet available as the Euclidean formulation of the theory is very difficult.

Besides many other models the question of dynamical supersymmetry breaking has been answered for $N = 1$ SYM using the Witten index [1] and low energy effective Lagrangian calculations [2]. Different instanton calculations [3–5] as well as the concept of Wilsonian low energy effective actions [6] agree with the scenario of unbroken supersymmetry. But all these calculations have a conceptual

problem in common: Supersymmetry breaking as a hysteresis effect cannot be studied, as explicit supersymmetry breaking is impossible to include (the notion of hysteresis effects in quantum field theories is discussed in Sect. 2). Consequently many spontaneous effects have to be introduced by assumption, which could anticipate the (non-) existence of dynamical supersymmetry breaking. The assumption of unbroken supersymmetry has important influence on the picture of superstring theory: Directly as in certain limits flat field theories appear on the branes of string theories and indirectly by establishing duality arguments within supersymmetry.

In this work we want to discuss once more the question of dynamical supersymmetry breaking from a very fundamental point of view. First we introduce the basic concepts used to determine the vacuum structure of quantum field theories (hysteresis effects) and apply them to $N = 1$ SYM. Such hysteresis effects have already been studied for $N = 1$ and $N = 2$ SYM in [7, 8]. But these results are incomplete as well: First it is difficult to compare them with other calculations as a non-standard structure of the QCD vacuum plays an important role and second the geometric approach used therein is known to be problematic in quantum effective actions.

The second part of this paper is devoted to the discussion of this last point: the relevance of superspace geometry in the context of quantum effective actions and its connection to other formulations like Wilsonian low energy effective actions or effective Lagrangians. It is of main importance as from our point of view only the quantum effective action can tell us the correct ground state. A justification of the geometric approach for effective Lagrangians or Wilsonian effective actions must thus be de-

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rived therefrom. Our result suggests that this cannot be done: A consistent description of the quantum effective action is found together with non-semiclassical effects only. These non-semiclassical effects can break supersymmetry and demand a new interpretation of superspace geometry. An alternative scenario is possible: Supersymmetry is unbroken but has a phase transition in the variation of the gluino mass at $m = 0$. We suspect that the resulting theory is highly infrared sick and probably does not exist at all. Of course these results are not free of assumptions either. As exact results are not obtainable we follow the philosophy of semiclassical analysis as far as needed to be able to make any statements. Especially we assume that we can define composite operators and their vacuum expectation values and that the unphysical fields from quantization need not be included explicitly.

In this paper we mainly want to collect known results from non-perturbative supersymmetry as well as non-perturbative QCD and classify them from the point of view of the fundamental question: “How do we find the true ground state of a quantum field theory?” We propose a new scenario for the dynamics of SYM, but this result is rather speculative. First steps towards a concrete model of this type will be presented elsewhere [9].

This paper is organized as follows: In Sect. 2 we review some basic facts about non-perturbative field theories and hysteresis effects. Here we formulate our recipe how to find the true ground state of a quantum field theory. Section 3 applies these ideas to $N = 1$ SYM and compares our ansatz with other low energy approximations known in the literature. In Sect. 4 constraints on dynamical supersymmetry breaking independent of our approach are discussed. It is shown in Sect. 5 that the standard interpretation of SYM theory is not compatible with all constraints on the dynamics found so far. A (rather speculative) way out of this is developed (Sect. 5) and discussed in Sect. 6. Therein we also comment on the Witten index and on alternative approaches to the low energy dynamics of SYM. Finally we give some comments about more complicated models (Sect. 7) and draw our conclusions (Sect. 8).

2 Non-perturbative QFT as thermodynamical limit

In this section we want to review some basic aspects of non-perturbative quantum field theory and discuss its relevance for a modern approach to a 4D QFT, where exact calculations in the non-perturbative sector are not available. Physical amplitudes are derived from the generating functional of the Green functions Z_M or from the heat kernel Z_E , written as path integrals:

$$Z_M[J] = \int \mathcal{D}\phi e^{iS_M(\phi, J)}, \quad Z_E[J] = \int \mathcal{D}\phi e^{-S_E(\phi, J)}, \quad (1)$$

where S_E is the Wick rotated action of S_M . At least formally we can define associated to every source extension of a classical action an effective action and an effective potential. These quantities are obtained by a Legendre

transformation with respect to the sources and the new variables are called classical fields:

$$Z_M[J] = e^{iW[J]}, \quad \frac{\delta W[J]}{\delta J(x)} = \tilde{J} = \langle \Omega | \phi(x) | \Omega \rangle_J, \quad (2a)$$

$$\Gamma[\tilde{J}] = \int d^4x \left(J(x) \frac{\delta W[J]}{\delta J(x)} \right) - W[J],$$

$$V_{\text{eff}} = \frac{1}{V^4} \Gamma[\tilde{J}] \Big|_{p \rightarrow 0}. \quad (2b)$$

The above definitions are given in Minkowski space; analogous definitions for Euclidean space follow straightforwardly. In the last equation V^4 is the space-time volume and the ϕ are the operators associated with the sources J . They may be basic field operators as well as composite ones. In the first case the effective action is often called the 1PI effective action, as it is the generating functional of the 1PI irreducible graphs. Not every source extension can lead to a well-defined effective action. The mapping $J \rightarrow \tilde{J}$ must be one-to-one and consequently the effective potential is always a convex function in the classical fields. We will discuss some of these problems when defining our supersymmetric effective action. If $\Gamma[J]$ exists, it defines thermodynamical relations between the sources and the spontaneous parameters of the associated operators. In the limit $V^4 \rightarrow \infty$ we have to turn off all sources in order to re-obtain our original theory and thus

$$\frac{\delta}{\delta \tilde{J}} \Gamma[\tilde{J}] = J(x) \rightarrow 0, \quad \frac{\delta}{\delta J(x)} W[J] = \tilde{J} \rightarrow \tilde{J}^*. \quad (3)$$

A non-vanishing value of \tilde{J}^* indicates the appearance of a spontaneous parameter (vacuum expectation value).

While the meaning of the path integral is well understood in perturbation theory, its interpretation in the non-perturbative sector is not straightforward. As we are trying to derive the properties of a quantum field theory from a classical expression like the action we can use the results of constructive field theory as a guide (see e.g. [10, 11] for a review of constructive quantum field theory). It is well known that we can solve a theory by introducing an UV and an IR regularization only. Many examples of perturbative and non-perturbative UV regularization schemes are known and it is an important feature of a well-defined theory that the resulting dynamics is independent of the regularization scheme. The issue of renormalization in the UV region is not a topic of our work. Rather we assume that it can be performed not only in perturbation theory but also in the non-perturbative region. Especially we assume that the definition of an operator in perturbation theory can be extended to the non-perturbative region and that we can give an interpretation of its vacuum expectation value.

The IR regularization plays a quite different role: In the context of perturbation theory a complete interpretation has been given by Bloch and Nordsieck [12] and by Weinberg [13], while it plays an essentially different role in non-perturbative dynamics, which we want to discuss in the remainder of this section. How should we choose the IR regularization? If the classical theory has a mass gap

we only need to restrict the trilinear and cubic interactions to a compact support, otherwise (as in gauge theories) we essentially have to put the theory into a finite volume. In the latter case we are confronted with the problem of choosing boundary conditions (BCs). We further have to distinguish fermions from bosons.

(1) The fermionic path integral is defined as the functional determinant of the corresponding operator. Therefore the eigenvalue problem has to be studied, which makes it understandable that even the infinite volume limit can depend on the BCs. An additional problem appears when regularizing the theory on topological spaces. Zero modes (instantons) then appear and the functional determinant is zero if at least one fermion is massless. We should remember that the definition of the partition function is only meaningful when attaching sources. Introducing sources η and $\bar{\eta}$ for the fundamental fermion fields the partition function in the presence of k zero modes ξ_i can be written as

$$Z[\eta, \bar{\eta}] = \prod_1^k \langle \bar{\eta} | \xi_i \rangle \langle \xi_i | \eta \rangle \quad (4)$$

$$\times \exp \left\{ - \int d^4x d^4y \bar{\eta}(x) G_e(x, y) \eta(y) \right\} \det'(i\mathcal{O}),$$

where $G_e(x, y)$ is the Green function on the space orthogonal to the zero modes and the prime indicates that the determinant is calculated over the non-zero eigenvalues only. A detailed but rather technical discussion of this problem has been given by Rothe and Schroer [14], see also [15]. Simple counting of zero modes shows that the chiral condensate of a theory regularized on the sphere or on the torus vanishes identically for $N_f > 1$ [16, 17]. This boundary effect does not have any physical meaning for the theory at infinite volume but is a wrong choice of IR regularization. The example shows how certain (not even exotic but very popular) BCs can lead to wrong conclusions if the thermodynamical limit is interpreted too naively. For further discussions and possible BCs solving the above problem we refer to different studies of the Schwinger model [15, 17–19].

(2) No simple interpretation of the path integral is available for bosons but it is said to be the “sum over all possible paths”. This is misleading in any model with degenerate vacua (e.g. due to symmetry breaking). This can be seen in simple examples as ϕ^4 in the Higgs phase or a one-dimensional ferromagnetic spin chain. The rotational symmetry is not anomalous and we can choose an UV regularization respecting the latter. Choosing BCs respecting the symmetry as well we get vanishing expectation values of the scalar field or of the spontaneous magnetization,

$$\langle \Omega | \phi | \Omega \rangle = \int \mathcal{D}\phi e^{iS} \phi = 0, \quad (5)$$

as the regularized path integral and the action are even under the symmetry while the field is odd. The simplest realization of this situation is the spin chain regularized on a circle. A straightforward interpretation of this effect exists: The theories do not have a single vacuum, but

infinitely many ones connected by symmetry transformations. To get a meaningful result one has to avoid integrating over all vacua, but one has to pick out one of them which is done by an appropriate choice of “boundary conditions” [11] (we put this in quotation marks to indicate that there are many ways to impose such a constraint).

Physical interpretations of this behavior can be given: In the presence of spontaneous parameters there exist phase transitions and indeed in this case boundary terms can get larger than volume terms. Considering the vacuum state in the limit $V^4 \rightarrow \infty$, the latter must be a pure state, while ground states in the finite volume are in general not pure. Thus the above “vacua” of the ϕ^4 theory are not acceptable. Although formally correct these interpretations of the phenomena fail to be applicable here: They assume that we know the vacuum expectation values (VEVs) of the Hamiltonian as well as of the basic field operators (in other words the redefinitions $\mathcal{H} \rightarrow \mathcal{H}'$ and $\phi \rightarrow \phi'$ such that $\langle \Omega | \mathcal{H}' | \Omega \rangle = 0$ and $\langle \Omega | \phi' | \Omega \rangle = 0$). If this is known we can choose one such $\phi' = \phi - \langle \Omega | \phi | \Omega \rangle$ and the scalar path integral is extended over the dynamical part ϕ' , only. In this work however we would like to use the thermodynamical limits to *determine* the above shifts and therefore these interpretations can be given a posteriori only. Different thermodynamical limits (i.e. infinite volume limits starting from different finite volume preparations of the system) are therefore treated independently although the final result may give a definite interpretation of the “wrong” limits in terms of the limits actually leading to the correct ground state. How can we read off the correct vacuum expectation values? The idea is rather simple: At least one limit (or one class of limits defined up to symmetry transformations) does lead to the correct ground state. The latter is defined to be the absolute minimum of the effective potential. Therefore we have to calculate the limits from all possible perturbations (we call these states trial vacua $|0\rangle$) and pick out the one(s) that minimize the effective potential:

$$|\Omega\rangle = \min_{V_{\text{eff}}} \{|0\rangle\}. \quad (6)$$

We should give some additional comments: We noted above that the effective potential is a convex function and thus there can exist only one minimum for it. This is true for one particular set of sources and BCs. Here we are speaking about *all possible* perturbations and they can lead in principle to infinitely many different convex functions, each having some minimum. These different minima need not be physically equivalent but are exactly the trial vacua $|0\rangle$. In this generalized sense the effective potential (being the set of all possible convex effective potentials) can now have more than one minimum and the correct ground state is indeed the absolute minimum with respect to the energy.

The perturbations can be boundary conditions or global sources [7, 8, 20, 21]. In the latter case sources are no longer seen as spatial restricted perturbations (typically as δ functions) but are extended over a large part of space-time, though the boundary condition $J(x) \rightarrow 0$ for $x \rightarrow \infty$ still holds. The sources can then be seen as

new coupling or mass parameters of the theory that have the following special features: First their value is not fixed (to obtain the original theory they have to be turned off in the end) and second, an associated classical field can be defined. A simple realization can be seen as follows [7, 21]: We split the finite volume V into a part V_{sub} and $V \setminus V_{\text{sub}}$. $V \setminus V_{\text{sub}}$ shall contain the whole boundary. Now we can choose the sources non-vanishing but constant inside V_{sub} and vanishing outside. The thermodynamical limit is taken as $V_{\text{sub}} \subset V \rightarrow \infty$ and the sources are getting relaxed in the end.

Using the concept of global sources the effective action can depend formally on both, the sources and the classical fields: In the above limit the sources are constant, $J(x) = J_0$, in (almost) the whole space-time and the procedure can be seen as a choice of new boundary conditions $J(x) \rightarrow J_0$ ($x \rightarrow \infty$). The variation with respect to the source is then a small perturbation on its constant part. Defining $J = J_0 + \Delta J(x)$, J_0 is irrelevant in the process of the variation ($\delta/\delta J(x)W[J] = \tilde{J}(x)$). But of course the classical field itself depends on the value of J_0 . We can then see the effective action as a function of the classical field and the constant source $\Gamma = \Gamma[\tilde{J}; J_0]$, where \tilde{J} itself depends on J_0 . Of course this is much the same as introducing a coupling constant J_0 and attaching a local source ΔJ to the same operator as J_0 and the dependence on the static part of the source plays a similar role as the dependence of the effective action on any coupling constant. Nevertheless we use the notion of global sources to keep track of the role of all parameters.

Global sources are especially useful if they generate a classical mass gap inside the perturbed region and make BCs (at least for the corresponding fields) irrelevant. As an example the above mentioned problem of vanishing chiral condensates can be solved without introducing different BCs but by attaching a global source having the effect of a fermion mass.

We should be careful with the limits of this prescription: We can always find the correct minima of the potential but the corresponding effective action needs not describe the correct dynamics. This can e.g. happen if a theory with instanton-like effects is perturbed in such a way that all or some instantons are getting suppressed. Our theory may be non-Hamiltonian, too. In this case we will find more than one ground state (up to symmetries) and the actually chosen state will depend on an external parameter.

Of course this program can be realized in principle, only. But there is a simple way to extract the relevant perturbations: To make the breaking of a symmetry visible, we need a trigger of the latter in the IR regularized theory. Therefore the interesting perturbations break the symmetries in question. In analogy to a spin system we call a spontaneous parameter, associated with such a perturbation, a hysteresis effect. Further restrictions on the choice of sources arise from the renormalization procedure: Symmetry invariance or covariance of the classical system including all sources must be extendible to the quantum theory. Otherwise the corresponding symmetries are not

realized on the level of the effective action. Moreover the perturbation must hold two stability conditions. First the resulting trial vacuum should be stable in a renormalization group analysis. This means that the classically trivial relaxing of the sources has a meaning in quantum theory: when turning the classical sources off the quantum system tends towards (and finally reaches) the original system. In addition a Minkowskian theory can have unstable potentials. Indeed the vacuum-to-vacuum transition probability $|\langle \Omega(t = +\infty) | \Omega(t = -\infty) \rangle|^2 = |e^{iW}|^2 = e^{-2\text{Im}W}$ is < 1 if the effective potential is complex, leading to a decay of the vacuum. Thus the effective potential must be real at any point of the perturbed system. If (at least in some range of the source parameters) the above conditions are met and the renormalized quantities stand in a one-to-one correspondence to the classical ones we can freely replace (in that range) the classical parameters by the renormalized ones.

As an application of this principle we note that it determines the value of θ in any QCD-like theory uniquely. This has been studied in detail in the appendix of [8] using the instanton picture. We will see that S^4 or T^4 as regularization spaces of the heat kernel or the generating functional of SYM are not sensitive to supersymmetry breaking. However the arguments given in [8] straightforwardly extend to any regularization space where θ has a non-trivial meaning as well as to perturbations by fermion masses (note that you have to introduce sources to both operators $\bar{\psi}\psi$ and $i\bar{\psi}\gamma_5\psi$). This is especially true when using local boundary conditions (e.g. ‘‘bag’’ BCs [17, 19]) or in a quantization on the light cone [22].

3 The effective action of $N = 1$ SYM

We want to apply the program sketched above to $N = 1$ SYM. The Lagrangian is given by

$$\mathcal{L} = \frac{1}{8C(G)} \left(\int d^2\theta \tau \text{Tr} W^\alpha W_\alpha + \text{h.c.} \right) + \text{GF} + \text{ghost},$$

$$W_\alpha = -\bar{D}^2 (e^{-V} D_\alpha e^V), \quad (7)$$

with the prepotential V used to quantize the theory in superspace. We work in Minkowski space with the generating functional

$$Z[J] = \int \mathcal{D}\phi e^{i(S_0(\phi) + S_J(\phi, J))}. \quad (8)$$

To decide whether supersymmetry is broken dynamically or not we introduce a set of global sources that

- (1) break supersymmetry as well as chiral symmetry;
- (2) connect the supersymmetric theory with some configuration where other dynamical effects (confinement, glueball) are (though not understood) well accepted;
- (3) could still be sensitive to the special geometry of supersymmetric theories.

The above conditions are satisfied by the concept of local couplings, where the coupling constant is replaced

by a chiral superfield [7]. We define a quantum effective action

$$\Gamma[\tilde{J}, \tilde{\bar{J}}] = \int d^4x \left(J(x) \frac{\delta W[J]}{\delta J(x)} + \text{h.c.} \right) - W[J, \bar{J}], \quad (9)$$

where $J = \tau + \theta\eta - 2\theta^2 m$ is the local coupling superfield. $\Gamma[\tilde{J}, \tilde{\bar{J}}]$ and $W[J, \bar{J}]$ are connected by thermodynamical equilibrium conditions and in the thermodynamical limit the effective action obeys the (anomalous) Ward identities [7, 8]. The chiral source field defines a set of three dual fields \tilde{J} . Its components are the VEV of the Lagrangian, of the gluon condensate and of a spinor, which represents the goldstino in case of broken supersymmetry. The following assumptions have to be made to be able to discuss supersymmetry breaking in a similar way to Veneziano and Yankielowicz [2]: The above effective action exists at least in its static limit and therein the classical fields \tilde{J} can be re-combined to a chiral superfield obeying the standard supersymmetry transformation rules. We would like to make some comments on this.

(1) The gluino condensate is certainly a natural perturbation for use to study dynamical supersymmetry breaking. The latter is expected to be connected to other dynamical effects of which chiral symmetry breaking is the only one accessible directly. Nevertheless other or additional breaking terms can be introduced at the classical level. Renormalization group analysis however suggests that such hard supersymmetry breaking terms are forbidden due to instabilities of the supersymmetric solution [23–27]. Although this is not of main interest in this context we would like to note that the same is true for possible gauge-symmetry breaking terms [28–30].

(2) Once we have identified the gluino term as the only reasonable perturbation, we can try to construct a chiral field from the classical variables having all the properties required. In perturbation theory such a field, the anomaly multiplet, in fact exists. It has been constructed in the Wess–Zumino model and in SQED and its existence has been proven in the non-Abelian case [31–34].

(3) In perturbation theory $N = 1$ SYM with a local coupling constant has been studied recently [35, 36]. This author finds an anomalous breaking of supersymmetry, as the conditions

$$\mathcal{S}(\Gamma) = 0, \quad \int d^4x \left(\frac{\delta}{\delta\tau} - \frac{\delta}{\delta\bar{\tau}} \right) \Gamma = 0 \quad (10)$$

cannot be satisfied simultaneously if the coupling is space-time dependent (\mathcal{S} denotes the Slavnov–Taylor operator), but there appears an anomaly in one of the above identities. If the anomaly is put into the Slavnov–Taylor operator the simple notion of superfields is lost. But we can put the anomaly into the $(\tau-\bar{\tau})$ identity as well. Then superspace is still valid and we can expect that the effective action is an integral over the standard superspace. A more detailed discussion of the relevance of this work for the conclusions of this paper must be postponed to a future publication.

3.1 Quantum or Wilsonian effective action?

Besides other models $N = 1$ and $N = 2$ Yang–Mills theories without [2, 37] and with matter fields [38–42] have been studied using the concept of Wilsonian low energy effective actions (LEEAs) or in the first case of a low energy effective Lagrangian. In combination with instanton calculations these concepts have been extremely successful to explore the non-perturbative region of supersymmetric gauge theories. As our comments on and our criticism of these concepts do not rely on the details of the results we do not want to repeat them at this place. Besides the original works cited above the results have been summarized in several review articles and lecture notes [43–49]. The motivation to use LEEAs instead of quantum effective actions (QEAs) is twofold: The authors would like to have an expression local in the fields, representing all relevant dynamics at low energies and they assume that the superspace can be reconstructed on the level of these fields completely. The low energy dynamics can then be written as

$$\mathcal{L}_{\text{eff}} = \int d^4\theta K(\Phi, \bar{\Phi}, J, \bar{J}, \Lambda) + \left(\int d^2\theta W(\Phi, J, \Lambda) + \text{h.c.} \right), \quad (11)$$

where Φ represents the quantum fields, J the local coupling and Λ the scale of the Wilsonian action. Local couplings and scale explicitly appear in the LEEA only and are treated as background fields. In both formulations any additional parametrical dependence (not expressible as an integral over superspace) is excluded. The main restriction on the above form is the holomorphic dependence of the superpotential on its fields. Thus every field appearing in the superpotential must depend on other fields in a holomorphic way as well. The kinetic term of the gauge fields is usually written as a chiral integral, but in contrast to the superpotential the latter is not irreducible and thus the non-renormalization theorem does not hold for this term. Nevertheless the effective kinetic part in the low energy approximation is written as a chiral integral and the holomorphy restriction is imposed. It follows that the β -function of SYM must be a holomorphic function. This condition does not apply to QEAs, which should not surprise, as the renormalization scheme itself does not have such a holomorphy constraint for the renormalized coupling constant. To escape this problem Shifman and Vainshtein introduced the notion of Wilsonian low energy effective actions within supersymmetry [6, 50]. Indeed the coupling constant of the LEEA differs from the one of the QEA by renormalization effects. The authors come to the conclusion that these effects turn the non-holomorphic coupling constant of the QEA into a holomorphic one of the LEEA. Further discussions of this effect have been given by Dine and Shirman [51]. The same result was obtained by Arkani-Hamed and Murayama [52] using a different picture than Shifman and Vainshtein. Although we do not agree with the treatment of the vacuum angle that serves as an example for $N = 1$ SYM in [50], we insist that

non-locality and the non-holomorphic dependence are crucial characteristics of QEAs.

The first important observation leading to results different from the ones cited above is the following: In our opinion we have to adjust the construction principle to the QEA and not the other way around, though a semiclassical ansatz for the QEA may be more difficult to find. As pointed out in Sect. 2 we have to study the hysteresis curve of explicitly broken supersymmetry back to the supersymmetric point and the natural formulation of this program is the QEA, while the above described LEEA does not help us in this situation (though the low energy effective Lagrangian of Veneziano and Yankielowicz is conceptually different from the Wilsonian LEEA it suffers from the same problem; this will become clear in the discussion of Sect. 5). Whether there exists a holomorphic coupling constant allowing the formulation in the form of a LEEA must be answered after the true ground state has been found using the QEA. We should not expect that this is possible: From the perturbative analysis of SYM with local coupling constant [35, 36] it has been found that the origin of the non-holomorphic dependence of the β -function is essentially different from the propositions in [6, 50] and [52] and that a simple redefinition of the coupling constant as proposed in the latter works cannot lead to a holomorphic coupling constant. In the language of [35, 36] Shifman and Vainshtein assume that the LEEA can be formulated using invariant counterterms only. Indeed in this case the β -function is strictly 1-loop and the coupling constant holomorphic. But it is not evident why the invariant counterterms should play a preferred role. Shifman and Vainshtein argue that all effects from non-invariant counterterms are IR divergences and are thus regularized in the LEEA. Kraus does not come to the same conclusion.

Let us assume for a moment that the point of view by Shifman and Vainshtein is correct. Why should we then look at the LEEA instead of the QEA? Following Shifman and Vainshtein we should look at the LEEA as this quantity alone is free of infrared subtleties. In the original paper introducing this concept [6] this is rather seen as a trick to obtain a holomorphic β -function, the LEEA is not seen as a physical object. In [50] the authors revised this opinion and they concluded that the objects (especially the value of the coupling constant) from the LEEA are physical, in contrast to their counterparts from the QEA. From our point of view this is a misunderstanding of the infrared problem of these theories. The serious infrared problem in perturbation theory is (hopefully) an effect of a wrong expansion and is getting removed in the non-perturbative region by the dynamical formation of a mass gap, a fact that Shifman and Vainshtein implicitly have to assume as well. There exist exactly two possibilities for the non-perturbative behavior of the theory.

(1) The IR problem is getting solved. Then we can freely remove any IR regulators and there exists no conceptual reason to prefer the LEEA (or any other IR regularized formulation) compared to our QEA, but our discussion shows that we are forced to use the QEA (or an equivalent formulation including the full dynamics): Such a

formulation alone can show how the IR divergences are getting removed, i.e. which symmetries survive this procedure and which are broken dynamically. By introducing an arbitrary IR regulator, Shifman and Vainshtein remove the relevant part of the dynamics by hand and thus miss an interesting point in the discussion of supersymmetry breaking.

(2) The infrared problem is not getting solved. Then indeed the QEA is ill defined but the LEEA is useless as well as the underlying QFT does not exist at all. Clearly we have to exclude this possibility by assumption.

We can now give a more detailed formulation of the assumptions made for our QEA.

(1) Like Veneziano and Yankielowicz we assume that all relevant low energy degrees of freedom are represented by the fields dual to J , i.e. the Lagrangian itself, the gluino condensate and the would-be goldstino in the case of spontaneous supersymmetry breaking.

(2) We assume that the explicitly broken theory with massive gluinos has a low energy behavior similar to QCD and that it does not undergo any phase transitions when varying m .

(3) The effective action defined in this way is a supersymmetric extension of the 2PI effective action constructed in perturbation theory by Cornwall, Jackiw and Tomboulis¹ [53–55]. In order to avoid misunderstandings we shortly want to comment on the relevance of this work for our construction: To get a perturbative approximation to the quark potential a bi-local source $\int d^4x d^4y \bar{q}(x) K(x, y) q(y)$ is introduced and the effective action in the presence of this source is calculated directly. As a result a local (chiral symmetry breaking) minimum is found, but the effective action is not bounded from below, but falls off to $-\infty$ as the dual field to the source is going to infinity. Using this effective action in our recipe for finding the physical minimum would lead either to the conclusion that the situation is unstable, or that our procedure is not applicable. But this is incorrect: Our procedure insists on the QEA being the Legendre transform of the energy functional $W[J]$ as given in (2) and thus being convex. Direct calculations of the QEA need not lead to convex functions: Several minima can occur and the function needs not be bounded from below. Before such a QEA can be used in our procedure the convex shell has to be taken, removing in our case the instability. Indeed the local minimum of the QEA by Cornwall, Jackiw and Tomboulis is physical, while the instability stems from the non-locality of the source [56].

(4) Within the restrictions already discussed the QEA should then be well defined for finite m . The defining fields are the classical fields of the Lagrangian multiplet $\tilde{J} \sim \langle \Omega | (1/8C(G)) \text{Tr} W^\alpha W_\alpha | \Omega \rangle$ and we assume that superspace can be reconstructed on these three components at least for the local part in the static limit. To distinguish

¹ The author would like to thank J.-P. Derendinger for drawing his attention to this work

this superfield from the set of its components we will refer to it as Φ :

$$\begin{aligned}\Phi &= \varphi + \theta\psi + \theta^2 L, \quad \varphi \sim \langle \Omega | \lambda\lambda | \Omega \rangle, \\ \langle \Omega | \mathcal{L} | \Omega \rangle &\sim \tau L + \text{h.c.}\end{aligned}\quad (12)$$

$N = 2$ SYM shows explicitly that the local part of the QEA derived this way is unacceptable as a *dynamical* result [8].

Although the theory can now be formulated using the dual fields only, it is useful to re-introduce some sources as discussed in Sect. 2. This is specifically done for the sources breaking the symmetries in question, as the trigger term is the constant source going to zero in the relaxing limit, while the value of the spontaneous parameter is an unknown quantity. For $N = 1$ SYM this trigger is the gluino mass and thus $\Gamma[\tilde{J}, \tilde{J}]$ is replaced by $\Gamma[\tilde{J}, \tilde{J}; m_0, \bar{m}_0]$ where m_0 is the constant part of the source m . As the mapping of the dual field onto its source must be one-to-one in the region where the effective action is well defined, we can freely replace the dependence on the dual field by a dependence on the re-introduced source. But this dependent variable is not a function of its dual field only, but can depend on all dual fields even in a non-holomorphic way (remember that it plays the role of a coupling constant; renormalization of the latter need not respect holomorphy as our source extension is not restricted by a non-renormalization theorem). In contrast to the LEEA of (11) we thus neither assume that superspace can be reconstructed on the level of the three possible sources τ , η and m , nor do we require a holomorphic dependence of the superpotential on these three parameters. Instead all quantities depend parametrically thereon.

Note that in our concept the effective action is now a function of the classical fields (constrained by geometry) and of the static part of the sources (having a parametrical dependence and including the YM coupling constant τ). But this dependence is defined in terms of a single source multiplet. There exists the possibility to define two source multiplets, one used in the Legendre transformation and the other one used as independent variable. Now the effective action is a functional of local classical fields as well as of local sources. This conceptually different ansatz has been discussed in [55].

(5) In the limit of vanishing gluino source m , our concept of global sources is problematic as

$$(\delta/\delta\tau(x))W|_{\tau \rightarrow \text{constant}} = 0$$

is true for any value of τ if supersymmetry is unbroken. This just represents the fact that unbroken supersymmetry for a coupling constant τ means unbroken supersymmetry for $\tau + \delta\tau$, too. Therefore we have to relax τ to its quantum-mechanical value before relaxing m . But this condition is not new as exploring the hysteresis line means that we relax the source which breaks the symmetry in question (in our case m) in the very end.

(6) Besides the ones discussed above, other problems of the QEA especially dangerous to supersymmetric theories

have been raised (see e.g. [43]). We can just stress again the following points: It is absolutely necessary to allow for explicit supersymmetry breaking terms regardless of any unbeloved consequences on the geometry of the theory. Moreover we have already pointed out that we should use this procedure to find the minima only. Indeed we are not able to show that some candidate for the true ground state found this way is unique, and we can thus never expect that our QEA captures the whole dynamics over this ground state correctly.

As final remark of this section we would like to mention the analogy of our proposals to QCD: In analytic calculations LEEAs and low energy effective Lagrangians have not been successful to determine the vacuum structure of QCD but their success relies on the fact that the vacuum is known from experiments. We think that this order (first the vacuum, then the low energy approximation) is crucial for any theory with a non-perturbative sector that is not available for exact calculations.

4 Constraints on dynamical supersymmetry breaking

We want to leave for a moment the construction principles of our effective action and discuss some constraints on dynamical supersymmetry breaking independent of the problems mentioned above. The first point are current algebra relations that lead to the postulation of a massless goldstino if supersymmetry is spontaneously broken and give a constraint on the value of the vacuum energy. If supersymmetry is unbroken the covariant Hamiltonian and its expectation value with respect to an arbitrary state $|\psi\rangle$ and to the ground state $|\Omega\rangle$ are given by

$$\begin{aligned}\mathcal{H} &= \frac{1}{2N} \sum_i (\{Q_1^i, \bar{Q}_{1i}\} + \{Q_2^i, \bar{Q}_{2i}\}), \\ \langle \psi | \mathcal{H} | \psi \rangle &\geq 0, \\ \langle \Omega | \mathcal{H} | \Omega \rangle &= 0,\end{aligned}\quad (13)$$

where N is the number of supersymmetries and Q_α^i is the supercharge of the i th supersymmetry. If supersymmetry is broken the supercharges are no longer well defined. For a single supersymmetry with supersymmetry current S_μ , the local version of the above relation leads to the famous order parameter of supersymmetry breaking [57]:

$$\begin{aligned}\int d^4x \partial^\mu \langle \Omega | T S_{\mu\alpha}(x) \bar{S}_{\nu\dot{\beta}}(0) | \Omega \rangle &= 2\sigma_{\alpha\dot{\beta}}^\rho \langle \Omega | T_{\nu\rho} | \Omega \rangle \\ &= 2\sigma_{\alpha\dot{\beta}}^\rho \epsilon_0,\end{aligned}\quad (14)$$

and $\epsilon_0 = 0$ means unbroken supersymmetry, $\epsilon_0 > 0$ spontaneously broken supersymmetry, while $\epsilon_0 < 0$ would signal a supersymmetry anomaly. Unfortunately the (perturbative) quantization of gauge theories destroys (13) and (14): The supercharge of the quantized theory is not time-independent and the Hamiltonian is not expressible in the form (13) [58, 59]. A time-independent charge is found after projecting onto the physical Hilbert space only. At the

moment we are not able to decide whether the positivity property of ϵ_0 survives the perturbative quantization or not. Green functions with one or more insertions of the supercurrent have been studied recently [60–62] but the verification of constraints on supersymmetry breaking from (14) is not yet possible [63]. This uncertainty shows the relative importance of all standard arguments about dynamical supersymmetry breaking as well as our discussion. We will assume in the following that at least after projecting onto the physical Hilbert space the positivity constraint still holds. Within the context of our work this assumption is certainly justified: If supersymmetry is really unbroken in perturbation theory, the fundamental relations of its algebra must be realized at least on the physical Hilbert space. If this were not the case, a completely new understanding of supersymmetry would be necessary. Moreover we follow the standard assumption that the unphysical fields introduced by the quantization do not contribute to the spontaneous parameters, i.e. operators including them have vanishing VEVs. For SYM, (14) together with the trace anomaly then leads to

$$\langle \Omega | T^\mu{}_\mu | \Omega \rangle = -\frac{\beta}{g} \langle \Omega | \mathcal{L} | \Omega \rangle = 4\epsilon_0 \geq 0, \quad (15)$$

and the VEV of the Lagrangian becomes the order parameter of supersymmetry breaking. The fact that the VEV of the Lagrangian must be positive to enable supersymmetry breaking is a severe constraint on the spontaneous parameters of this theory. As a side-remark we want to note that in our approach supersymmetry cannot be broken directly by a gluino condensate as the latter is the lowest component in the defining superfield.

4.1 Supersymmetry and the sign of $\langle \Omega | F_{\mu\nu} F^{\mu\nu} | \Omega \rangle$

Following our assumption that explicitly broken SYM has a similar vacuum structure as QCD, the number of spontaneous parameters seems to reduce to $\langle \Omega | \text{Tr} F_{\mu\nu} F^{\mu\nu} | \Omega \rangle$ and $\langle \Omega | \text{Tr} \lambda \lambda | \Omega \rangle$. The remaining operators in the Lagrangian should have vanishing VEVs and by the assumption of a smooth dependence on m this should hold at the supersymmetric point too. Thus (15) reads $\langle \Omega | \text{Tr} F_{\mu\nu} F^{\mu\nu} | \Omega \rangle \leq 0$, which is a remarkable result. Completely independent of supersymmetry we can ask whether there exists a constraint on the sign of F^2 and all arguments suggest the same result: $\langle \Omega | \text{Tr} F_{\mu\nu} F^{\mu\nu} | \Omega \rangle \geq 0$ and supersymmetry breaking seems to be excluded as the trivial result $\langle \Omega | F^2 | \Omega \rangle = 0$ remains only. For completeness we would like to list some of the arguments.

(1) *Sum rules.* Based on the work by Shifman, Vainshtein and Zakharov [64, 65] the value of $\langle \Omega | (F^2)_{\text{QCD}} | \Omega \rangle$ has been estimated to be about 0.250 GeV^4 (see e.g. [66] for recent results on this topic).

(2) *Non-decoupling theorem.* If the theory smoothly depends on the gluino mass, we can study the limit $m \rightarrow \infty$. Indeed the trace anomaly leads to an interesting relation (\mathcal{L} still represents the SYM Lagrangian of (7)):

$$T^\mu{}_\mu = -\frac{\beta}{g} \mathcal{L} + \left(\frac{m}{2C(G)} \text{Tr} \lambda \lambda + \text{h.c.} \right), \quad (16)$$

$$\beta(g) = -\beta_{\text{YM}}(g) + \beta_\lambda(g), \quad \beta_{\text{YM}} > 0, \quad \beta_\lambda > 0.$$

Imposing the constraint that in the limit $m \rightarrow \infty$ the trace anomaly reduces to the known result of pure gluon dynamics and taking the vacuum expectation value we get

$$\frac{\beta_\lambda(g)}{4g^2 C(G)} \langle \Omega | \text{Tr} F_{\mu\nu} F^{\mu\nu} | \Omega \rangle$$

$$= -\lim_{m \rightarrow \infty} \left(\frac{m}{2C(G)} \langle \Omega | \text{Tr} \lambda \lambda | \Omega \rangle + \text{h.c.} \right). \quad (17)$$

Of course this relation is only meaningful if SYM indeed tends towards gluon dynamics in this limit. There is in fact a simple constraint on this relation stemming from the vacuum angle: Thermodynamical restoration of CP violation [8, 20, 67, 68] leads in SYM with a gluino mass to the following constraints [8]:

$$(\vartheta - \vartheta_V) + \arg m = 0,$$

$$m \langle \Omega | \text{Tr} \lambda \lambda | \Omega \rangle = \bar{m} \langle \Omega | \text{Tr} \bar{\lambda} \bar{\lambda} | \Omega \rangle. \quad (18)$$

The fact that the resulting gluon dynamics must have $(\vartheta - \vartheta_V) = 0$ tells us that only real gluino masses can lead to smooth decoupling, or else the vacuum angle ϑ_V makes a jump. From the second relation we see that in this case the condensate must be real. In the limit of a heavy mass the expectation value of F^2 thus has an opposite sign of the expectation value of the gluino condensate. The latter sign is negative in analogy to QCD (this already follows from PCAC analysis [69]; for a discussion within QCD see e.g. [16, 70]). The notion of decoupling a particle by making its mass heavy is intuitively pleasing, but it is of course very difficult to make exact statements about the behavior of the remaining degrees of freedom. Comparing the situation again with QCD the non-perturbative region could be crucially different in the latter case: While in QCD fractional winding numbers are excluded, they are not in pure gluon dynamics (YM theory in the following) as well as in SYM. The relevance of fractional winding numbers for non-perturbative effects is a highly non-trivial problem. It is beyond the scope of this paper to discuss this problem in detail, but the example again illustrates the importance of BCs. If fractional winding numbers are assumed to be relevant (see e.g. [16]) a smooth decoupling of QCD towards YM is endangered. In contrast to QCD SYM still decouples smoothly to YM and regardless of this important difference we have to assume that SYM with a gluino mass has a behavior similar to QCD. A different point of view has been discussed in [71]. Therein it is argued that fractional winding numbers are irrelevant in QCD as well as in YM theory. Then the above problems disappear at the price of a new problem at the other end of the mass scale: Now the ground state of SYM seems to be degenerate [1] leading to domain walls [72, 73]. This degeneracy is an effect of the specific choice of IR regularization and disappears with fractional winding numbers [16]. In the context of integer winding numbers the degeneracy is found to be lifted by a more involved study of the

thermodynamical limit, as the phase of the condensate is getting fixed by our program discussed in Sect. 2 [8, 74].

(3) *Euclidean background fields.* Stability conditions on constant gauge fields have been studied in [75, 76] and its significance as semiclassical ansatz for the YM vacuum has been discussed in [77, 78]. These authors study the heat kernel of Yang–Mills theories and therefore the constraints have to be understood in Euclidean space. Nevertheless it is worth mentioning the agreement of these results: Field configurations are stable if $\langle \Omega | \mathbf{E}_M^2 | \Omega \rangle = -\langle \Omega | \mathbf{E}_E^2 | \Omega \rangle \leq 0$.

(4) *Minkowskian background fields.* The study of Minkowskian background fields in gauge theories goes back to the work of Euler/Heisenberg [79] and Schwinger [80] on QED that led to an important result: If $F^2 < 0$ the potential is not only away from its minimum but it is unstable, i.e. the effective potential becomes complex. The generalization of this analysis to YM theories and QCD has been performed by Cox and Yildiz [81, 82]. Although non-Abelian gauge theories are much more complicated than QED we expect a complex effective potential for $F^2 < 0$ in the first case too.

5 Breaking supersymmetry with $\langle \Omega | \mathbf{F}_{\mu\nu} \mathbf{F}^{\mu\nu} | \Omega \rangle > 0$

Can we conclude that either supersymmetry is unbroken or that at least for small m the vacuum structure is not similar to QCD? We think that this conclusion is unwarranted. On the level of the field content there exists an important difference between QCD and SYM: the existence of auxiliary fields. They play an important role in the breaking mechanisms of supersymmetry.

5.1 The Lagrangian as auxiliary field and the limits of the geometrical approach

In the geometrical approach to the effective action there exist two different types of auxiliary fields: the auxiliary field of the classical field describing the effective action and the auxiliary field of the underlying quantum theory. We will refer to them as second- and first-generation auxiliary fields respectively. In the construction of [2, 7] the local part of the effective action, being expressed in terms of the Lagrangian or anomaly superfield Φ (see (12)), is of the form

$$\Gamma[\Phi, \bar{\Phi}; m, \bar{m}] = - \int d^4\theta K(\Phi, \bar{\Phi}; m, \bar{m}) + \left(\int d^2\theta W(\Phi; m, \bar{m}) + \text{h.c.} \right). \quad (19)$$

The effective potential then reduces to [7]

$$V_{\text{eff}} = \frac{1}{V^4} (-\bar{L} g_{\varphi\bar{\varphi}} L + (L W_{,\varphi} + \text{h.c.})). \quad (20)$$

If (19) should represent a meaningful Lagrangian in an expansion up to second order derivatives as in [2], $g_{\varphi\bar{\varphi}} > 0$

and the potential as a function of L is not bounded from below and does not even have a local minimum. Looking at the point $m = 0$ only, this is not surprising: L is the auxiliary field which has a definite interpretation within supersymmetry. The potential is getting maximized with respect to L and the remaining (physical) potential is positive semi-definite. The concave potential of the auxiliary field is harmless as long as the full effective action has no derivative terms acting thereon: Its equations of motion are algebraic and the negative semi-definite potential does not have a physical meaning in the sense of our discussion in Sect. 2. In contrast to a different interpretation of the auxiliary field introduced below we call this behavior non-dynamical. When studying a dependence on m however this behavior is particularly dangerous: Our extension of the system has been arranged in a supersymmetry covariant way for any finite m . In a naive application the above structure would be true even for large m and pure gluon dynamics would have a reasonable approximation in terms of an auxiliary field, i.e. its low energy approximation would not have any derivative terms at all. The ansatz would then be wrong for large m and thus for any finite m and according to our discussion it would be useless for studying supersymmetry breaking. Of course such a criticism of the work by Veneziano and Yankielowicz is – as it stands – not acceptable: The effective Lagrangian has been arranged for vanishing gluino mass and certainly a naive extrapolation to finite masses does not capture all dynamical effects that could take place in such a deformation. In the remainder of this section we want to argue that even a more careful treatment must lead to a similar conclusion.

We will focus on the possibility of unbroken supersymmetry at $m = 0$ which is the only scenario compatible with (20). At $m = 0$ thus $\mathcal{W}_{,\varphi} = 0$ and $\mathcal{W}_{,\varphi} < 0$ is possible for $m \neq 0$ leading to an acceptable VEV of F^2 . If (19) shall represent the full dynamics of the system our conclusion is certainly correct: (19) has a positive semi-definite convex potential in the physical fields for all values of m and the number of derivatives is restricted to 2. By its construction this effective action can never break down as the momentum expansion is always exact and the potential always stable – the discussion of the second statement is analogous to the discussion of this point in the more general model below.

We thus conclude that (19) does not represent the full effective action, but it is assumed that the effective potential (20) describes at least qualitatively the correct minima of the theory. This implies $g_{\varphi\bar{\varphi}} > 0$, otherwise the potential is either trivial or not bounded from below after eliminating the auxiliary field. In fact all other interpretations fail to be applicable: A non-trivial phase of $g_{\varphi\bar{\varphi}}$ would lead to an unstable potential and with $g_{\varphi\bar{\varphi}} < 0$ the potential for the gluino condensate is not bounded from below. Of course these strict conditions hold in the minimum only. Away from the minimum different complex phases may appear.

The geometrical effective potential is embedded in a more complex effective action including derivative terms

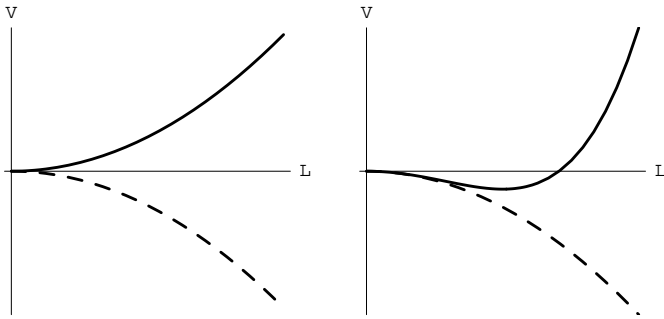


Fig. 1. Possible forms of a non-trivial potential of the classical Lagrangian. The new contributions can turn the maximum at $L = 0$ into a minimum (*left hand side*) or they can move the minimum away from the origin (*right hand side*). The dashed line represents the perturbative potential in L , $V = -\bar{L}g_{\varphi\bar{\varphi}}L$, the solid line the complete non-perturbative potential

and additional potential terms². We cannot specify their form but only some conditions: At $m = 0$ L is an auxiliary field, or else there are dominant contributions to the effective potential not included in (20). At $m \rightarrow \infty$ L must become a dynamical field and the potential must have a minimum in L with $L_0 < 0$. This change of the behavior of L implies the existence of a phase transition: The effective potential is always in its allowed region, i.e. we certainly have a real V_{eff} for all m defined over the range $0 \geq L \geq -\infty$. Whatever the (static or dynamical) part of the effective action is doing between $m = 0$ and $m = \infty$, if it should turn L from an auxiliary field into a dynamical field, the potential must at some point be completely flat. Even if this is thought to be too strong a conclusion in the given approximation the following points are certainly true: The potential is at some point zero at $L = \infty$ and there exists a region where it is (almost) flat around the maximum (turning into a minimum). This is completely sufficient to see that the system would be unstable. Thus we conclude that there exists a phase transition at some critical value of the gluino mass m_c . There are two qualitatively different ways how such a phase transition could look like as shown in Fig. 1 (remember the constraints $L \geq 0$ and $\text{Im}L = 0$). The new contributions to the potential above the phase transition can just turn the maximum into a minimum leading to a formally smooth value of L in the whole range of m or the value can make a jump at the phase transition. Despite the fact that L is smooth in the first case some other parameter has to make a jump, the example just shows that L need not be the order parameter distinguishing the two different phases.

² The importance of some dynamical arguments in the following does not stand in contradiction to the limited relevance of our effective action. If the extremization of the effective potential leads to a maximum in some field, the latter must be non-dynamical if the corresponding state plays any role in the true ground-state. Our effective action may be incomplete at $p \neq 0$ as we may have missed some physics not reachable by our extension. But this is not important here, as we only need to know that there *is* some dynamics

We stress that this conclusion is correct even if our effective action does not represent the theory for all gluino masses m (which will be important in Sect. 5.2). It could happen that the set of relevant classical operators is different for different regions of the gluino mass. But the question whether F^2 is dynamical or not is a problem of physics that must be represented correctly in all possible QEAs. Thus our conclusion is correct if F^2 is a relevant low energy degree of freedom for all m , which is included in our assumption of a QCD-like behavior.

Using again the comparison with QCD we expect the phase transition at $m_c = 0$. This possibility indeed exists in the analysis of [7] and cannot be excluded in our discussion. We just would like to stress the consequences thereof: The phase transition is associated with the spontaneous parameter of F^2 , a non-perturbative effect. Such a phase transition is particularly dangerous to all other non-perturbative effects, namely chiral symmetry breaking and confinement, which have been implemented by assumption. In fact, the solution in [7] suggests unbroken chiral symmetry which is consistent with [2]. Besides these more technical problems the system would be highly unstable and we do not see how this could still be an acceptable field theory.

In the alternative scenario a phase transition does not exist and additional contributions to V_{eff} are relevant even at $m = 0$. In particular the effective potential does not get maximized with respect to L , but minimized. This does in principle not stand in contradiction to L being an auxiliary field at $m = 0$, but opens new possibilities for supersymmetry breaking.

This discussion shows that indeed the effective Lagrangian by Veneziano and Yankielowicz suffers from a similar problem as the LEEA though it does not introduce a IR regulator. The meaning of a hysteresis curve in the context of this approach remains mysterious and thus it is not a suitable ansatz in the light of our considerations of Sect. 2.

5.2 The role of the fundamental auxiliary fields

We want to discuss some consequences of a scenario without phase transition. In this scenario there appear explicit derivative terms in the auxiliary field at least above some scale of the gluino mass and thus the second-generation auxiliary field L changes its character towards a physical field. Many points of this section are highly speculative and certainly more investigations are needed to develop a concrete model where the effects proposed in the following could be studied.

We have studied the second-generation auxiliary field without noting the possible importance of the first generation. Indeed more carefully the constraint derived from (15) reads

$$\frac{1}{C(G)} \langle \Omega | \text{Tr} \left(\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{1}{2} D^2 \right) | \Omega \rangle \geq 0, \quad (21)$$

and supersymmetry is broken if and only if the auxiliary field gets a non-perturbative VEV with $\langle \Omega | D^2 | \Omega \rangle >$

$(1/2)\langle\Omega|F^2|\Omega\rangle$. The fact that supersymmetry breaking is driven by the VEV of the auxiliary fields is an old wisdom from perturbation theory [83, 84]. As many restrictions on perturbative supersymmetry (by assumption) hold in the non-perturbative region too, the importance of the auxiliary fields therein is not surprising. Breaking supersymmetry by postulating non-trivial dynamics of the auxiliary fields does certainly not look very appealing, but under the given assumptions it is a correct and necessary proposition. We want to point out some restrictions and consequences of this scenario.

Non-trivial dynamics of the auxiliary field leads to a complete breakdown of the supersymmetry covariant approach: As the auxiliary field changes its character towards a physical field, the extremization of the potential must lead to a minimum and non-geometrical contributions are relevant. Moreover all supersymmetry covariant expressions (sources and BCs) depend on the combination $L = -(1/4)F^2 + (1/2)D^2$ only. Of course we can determine the minimum $\delta_L V_{\text{eff}}(L_0, \bar{L}_0; m, \bar{m}) = 0$. As L_0 is directly related to the goldstino coupling the covariant effective action has still a meaning at least for small masses m where the (pseudo-) goldstino is a special particle. Above this scale the combination loses its meaning and the physics is probably not described by these combinations any more. In contrast to the case with a trivial auxiliary field $\langle\Omega|D^2|\Omega\rangle = 0$ however, the goldstino coupling is not a primary object but we have to study F^2 and D^2 independently. Trying to impose a constraint on supersymmetry covariant objects only leads to difficulties: Infinitely many combinations of the gluon and the auxiliary field lead to a specific value of L (even for $L = 0$). In the IR regularization or in a semiclassical calculation this leads to a summation over all these combinations, and the VEV of a single operator F^2 or D^2 is no longer well defined. By treating the two operators as independent objects $L(m)$ (or the dependence of L on any other external parameter) describes a line in the F^2 - D^2 plane. Unbroken supersymmetry would imply that the line starts at the origin (supersymmetric point) and that F^2 develops a VEV as m increases. For broken supersymmetry the shape of the line is unknown. It starts at some point with $2D^2 > F^2$ and this constraint is fulfilled within the range of the pseudo-goldstino being a special particle. Above this range the combination L is no longer meaningful, and thus the line is unimportant (or perhaps not even defined). Besides $L(m)$ which could be calculated for small m by a chiral perturbation theory for the goldstino and in the large m limit by using YM results, independent knowledge of one of the two involved basic operators is needed. Finding this line would answer many open questions about dynamical symmetry breaking in SYM and must be one of the main areas of future research.

In this scenario supersymmetry breaking is a non-perturbative non-semiclassical effect: It can be established from an IR regularization mixing the physical fields with the auxiliary field, only (i.e. the separation of the path integral into a physical and an auxiliary field part is no longer possible). Clearly spaces allowing the definition of

instantons are not sensitive to non-perturbative effects of D^2 . Instanton calculations have been performed in different regions [3–5] and have found to be consistent with each other within the semiclassical approximation [85, 86]. In agreement with our discussion supersymmetry does not break by instanton induced effects.

Which are the spaces that make effects from auxiliary field visible? A simple analysis of the above separation condition shows that source extensions alone are useless. Some sources like the goldstino source couple the auxiliary field to the physical ones. But neither do they lead to derivative terms in the auxiliary fields, nor do they change the sign of the potential, as this requires effects from non-renormalizable operators. There is room for speculations within more general BCs, as non-renormalizable operators can now be included, but at the moment we are not able to suggest any concrete calculation that could test our proposition.

Do the auxiliary fields turn into physical fields completely or are they still non-dynamical in the end? If $\langle\Omega|D^2|\Omega\rangle \neq 0$ this can only be due to quantum fluctuations and there must be a finite correlation $\langle\Omega|D(x)U(x, y)D(y)|\Omega\rangle \neq 0$ at least for small distances. This all happens due to an infrared effect and thus at least in this region the auxiliary field is indeed a physical field. Two different interpretations are possible: The first-generation auxiliary field is non-dynamical regardless of the value of m after removing all IR regularizations. Thus we catch all its important effects by replacing D^2 by its VEV in the classical Lagrangian. This leads to an alternative interpretation of this constant: Hughes and Polchinski [87] have shown that (14) can consistently be generalized to

$$\int d^4x \partial^\mu \langle\Omega|T S_{\mu\alpha}(x) \bar{S}_{\nu\dot{\beta}}(0)|\Omega\rangle = 2\sigma_{\alpha\dot{\beta}}^\rho \langle\Omega|T_{\nu\rho}|\Omega\rangle + C, \quad (22)$$

where C is a dynamical parameter and exactly represents the VEV of the fundamental auxiliary field after its elimination.

In the second interpretation the first-generation auxiliary field remains dynamical in the thermodynamical limit. At the moment this is pure speculation and we cannot give any similar model where this would happen.

Two important points in our discussion are highly speculative.

(1) The first point concerns the new contributions to the effective action, which are not of the form (11) changing the potential of the second-generation auxiliary field and leading to derivative terms thereof. The existence of such contributions immediately raises the questions of the realization of supersymmetry on the level of the QEA and of the validity of a momentum expansion of the latter. It is beyond the scope of this paper to answer these questions here. A simple model of this type will be presented elsewhere [9]. We also want to note that within a quite different context the effect of turning an auxiliary field into a dynamical one is known: in effective actions of SQCD based on gauged non-linear sigma models [88–91]. We should be careful in drawing any conclusions from

this, but the effect itself shows that the application of superspace geometry is not at all straightforward.

(2) Moreover, it is not understood which role the stability constraint from Sect. 4.1 plays from the point of view of the fundamental theory. All arguments in this paper have been semiclassical and we do not know exactly to what objects the symbols $\langle \Omega | F^2 | \Omega \rangle$ and $\langle \Omega | D^2 | \Omega \rangle$ refer to. The dynamics and the VEV of the first-generation auxiliary field discussed in this section apply to the semiclassical object $\langle \Omega | D^2 | \Omega \rangle$ and at the moment we are not able to conclusively relate this object to any known characteristic of the underlying theory.

Finally we want to point out that the assumption of vanishing VEVs in the ghost sector does not contradict our proposition of a non-trivial D^2 : The quantization of gauge theories can be performed in many different ways and depending on the procedure different unphysical fields appear. The existence of the auxiliary fields in supersymmetry however is unambiguous in the classical and quantized theory.

6 Discussion of the $N = 1$ result

Before going into the discussion of our results we want to summarize the four different low energy behaviors of SYM that we found.

(1) Supersymmetry is unbroken.

(a) Neither first- nor second-generation auxiliary field receive non-perturbative contributions. This implies the existence of a phase transition in the variation of the gluino mass with $m_c = 0$. We expect that all condensates vanish and conclude that the theory does not have an acceptable infrared behavior.

(b) The first-generation auxiliary field can be eliminated consistently but the potential in L has a minimum due to non-perturbative contributions. For $m = 0$ the minimum must be at $L = 0$ and supersymmetry is unbroken.

(c) Neither first nor second-generation auxiliary field behave non-trivially, but the minimum for $m = 0$ is still at the supersymmetry conserving point.

(2) Both auxiliary fields get non-perturbative contributions and supersymmetry breaks dynamically.

For (1a) the structure of the vacuum has been discussed. Especially in the cases (1b) and (1c) the latter is unknown as the geometrical approach for effective Lagrangians does not lead to the correct results. From our point of view it would be very surprising if the auxiliary fields could get non-trivial contributions without breaking supersymmetry. Provided $N = 1$ SYM does exist as a quantum theory and can be described at low energies by the effective action defined in this work, the favorite for its low energy behavior is the scenario (2).

6.1 Dynamical supersymmetry breaking and the Witten index

When we want to give an interpretation of our discussion two questions arise: Although the LEEA or low en-

ergy effective Lagrangian approaches have serious conceptual problems, the result derived therefrom could anyway happen to be correct. Can we exclude this? Besides this semiclassical approximation other arguments for unbroken supersymmetry have been given. What is their relevance within our discussion?

One part of the first question has already been answered in the last section: We cannot exclude a phase transition at $m = 0$. The resulting theory has $\langle \Omega | F^2 | \Omega \rangle = 0$ and the analysis of [7] suggests $\langle \Omega | \lambda \lambda | \Omega \rangle = 0$ as well (this solution stands in agreement with [2], further discussions of this state have been given in [5]). As all spontaneous parameters vanish we expect that the theory is not confined either. Besides the instability to perturbations we do not expect the IR problem to get solved. Apart from this both solutions (the one of [2] as well as the one of [7]) are probably incomplete as they do not have the correct analytical structure. In [7] $F^2 < 0$ everywhere except at the origin; [2] has $F^2 < 0$ in the region between the chirally symmetric and the chirally broken minimum. None of the two generates a complex phase within these regions and the strict constraints of the geometry make it difficult to include this instability (notice that the metric $g_{\varphi\bar{\varphi}}$ must by its construction be independent of L).

Besides the LEEA we have discussed the instanton calculations and we do not want to go more into details. An important argument against supersymmetry breaking seems to be the Witten index [1]. We want to give a brief comment on the work by Witten. Three ingredients are crucial to come to the conclusion that supersymmetry must be unbroken: holomorphic dependence, independence of the boundary conditions and availability of perturbation theory. Due to the holomorphic dependence of the superpotential we only need to answer the question of supersymmetry breaking for one (non-vanishing) value of the coupling constant (the latter is of course chosen to be small). Now Witten explicitly states that the index of an operator (and thus the fundamental characteristics of a trial vacuum state) is independent of the boundary conditions and that he thus is allowed to choose them arbitrarily. In this general form we do not agree with this (see Sect. 2). After choosing boundary conditions that at least allow one to do perturbation theory of the models in question, Witten argues that this is enough to calculate the index. To decide whether this simplification is allowed or not we have to clarify the meaning of the phrase “a theory in its perturbative region”: It can mean that perturbation theory gives a reasonable approximation to the true result, although the series need not converge, or it can mean that the series really converges. An example of the first kind of understanding is the vacuum of a non-Abelian gauge theory with small coupling constant. There are non-perturbative effects (chiral symmetry breaking, dynamical mass gap), but these effects are very small and thus the non-convergent perturbative expansion can nevertheless give a good approximation. It is often speculated that QCD in the deconfined phase is an example of the second kind. Indeed all known effects of non-perturbative dynamics seem to vanish and it could be that the perturbative

expansion now converges. Which kind of interpretation should we use if we want to show that supersymmetry is unbroken? Certainly the first one. Supersymmetry is unbroken if and only if the vacuum energy is *exactly* zero. For small coupling constants a (non-perturbative) supersymmetry breaking effect may be suppressed exponentially; this does not help as the holomorphy argument is immediately useless if the effect does not vanish completely. Certainly we cannot show that the perturbation series for supersymmetry with small coupling constant in a finite volume really converges.

Can the special structure of Witten's calculation using a version of the Atiyah–Singer index theorem justify a perturbative calculation? An argument has been given in [1] that this could be true: Consider an operator with non-zero index (say $n \neq 0$) in perturbation theory. If the index shall become zero non-perturbatively we cannot argue that the n states move away from zero a little bit: Supersymmetry only allows pairs of states with non-zero energy and thus the index remains n . Non-perturbative corrections therefore have to change the spectrum completely. Witten argues that such a correction would change the asymptotic behavior of the potential and he assumes that non-perturbative contributions cannot do this (at least for small coupling constants). We do not agree with this conclusion. The effect of non-perturbative contributions is exactly to change the potential in such a way: Confinement indeed changes the spectrum of the theory completely and we expect that this could also change the asymptotic behavior of the potential. Within the whole discussion the actual value of the coupling constant is completely irrelevant (as long as it is non-zero). This difference between the asymptotic behavior of the perturbative and the non-perturbative potential (for fixed value of the coupling constant) does not at all stand in contradiction to the fact that the non-perturbative potential is not allowed to change its asymptotic behavior when varying the coupling constant. These are two completely independent properties of the (perturbative and non-perturbative) potential. For additional discussions of related problems in a semiclassical analysis we refer the reader to [77].

Even if the Witten index would be a correct analysis of the theory within the special choice of boundary conditions, its consistent interpretation within our framework is quite easy: The author uses BCs that do not break supersymmetry. Thus we cannot expect that he will find a supersymmetry breaking state at large volume. Assuming that the state found in the limit is a reasonable candidate for the ground state, is there any argument that it must be the true ground state? We have seen that this is not true in general and – in contrast to a common misunderstanding of constraints from supersymmetry – the latter does not help in this situation. The supersymmetric trial vacuum minimizes the VEV of the energy-momentum tensor but (as a function of the classical fields) this is certainly not the correct quantity getting minimized by the true ground state (on the semiclassical level this has been discussed for YM in [77]). The analogy of the energy-momentum tensor and the effective potential holds in perturbation theory

due to the non-renormalization theorem, but the latter need not be valid in the non-perturbative region: Denoting by φ_0 the value of the fields at the minimum of the effective potential, the consequences of (14) for perturbation theory are

$$\langle \Omega | T^\mu{}_\mu | \Omega \rangle \geq 0 \Rightarrow V_{\text{eff}}(\varphi_0) \geq 0, \quad (23a)$$

while a possible scenario of non-perturbative supersymmetry breaking is

$$\langle \Omega | T^\mu{}_\mu | \Omega \rangle > 0 \Leftrightarrow V_{\text{eff}}(\varphi_0) < 0. \quad (23b)$$

The VEV of D^2 shows in a very simple way how Witten's vacua become irrelevant: The minimum in the effective potential lies at $L_0 > 0$ and $V_{\text{eff}}(L_0) < V_{\text{eff}}(0)$, but clearly

$$\begin{aligned} & \langle \Omega | T^\mu{}_\mu(L_0) | \Omega \rangle \\ &= -\frac{1}{4} \langle \Omega | F_{\mu\nu} F^{\mu\nu}(L_0) | \Omega \rangle + \frac{1}{2} \langle \Omega | D^2(L_0) | \Omega \rangle \\ &> \langle \Omega | T^\mu{}_\mu(0) | \Omega \rangle. \end{aligned} \quad (24)$$

The effect can take place as the wrong sign from the classical potential remains in the energy-momentum tensor while it is changed in the effective potential by non-semiclassical contributions.

Though we do not agree with Witten's interpretation of his calculation, the index can nevertheless unravel interesting properties of dynamical supersymmetry breaking: If the index of a theory within a certain choice of BCs is found to be non-zero, supersymmetry can break dynamically if and only if one of the following points applies.

- (1) The full non-perturbative potential has a asymptotic behavior different from the approximation used to calculate the index.
- (2) There exist non-perturbative effects that do not respect the non-renormalization theorem and that destroy the perturbative equivalence between the vacuum energy and the minimum of the effective potential (cf. (23b)).

According to our present knowledge both effects are not only non-perturbative but also non-semiclassical.

6.2 Different results from different geometry?

All results of Sect. 5 are valid under specific assumptions on the properties of SYM under quantization, only. There still exists the possibility that the specific choice of the geometry is wrong and not the geometrical approach itself. We want to make some comments on this problem. The strategy of the approaches discussed in this work is to use solely gauge singlets as classical fields. Following the philosophy of semiclassical approximations not to include non-physical fields from the quantization of the theory, the source extension is then complete and unique. There exists exactly one superfield invariant under full SUSY-gauge transformations whose highest component is a candidate

for the classical action (if there would exist any other superfields, our classical Lagrangian would not be the most general one obeying all symmetries and we would have to include this superfield in the action). Moreover, there exists exactly one possibility to extend the coupling constant supersymmetry covariantly to a superfield. Thus we expect that all other source extensions at least partially break supergauge invariance. We can illustrate this on the basis of the simplest generalization of our extension: The chiral source extension is problematic as the fundamental structure of the action is not chiral. Thus it would be most natural to consider the action as an integral over full superspace and to introduce a full source multiplet. We call the new multiplets Φ_f and J_f with

$$(\Phi_f)|_{\theta^2\bar{\theta}^2} = L, \quad (J_f)| = \tau. \quad (25)$$

If the chiral extension is indeed inconsistent on the level of quantum operators, the operator superfield associated with Φ_f , Φ_f^{op} enters in some expression in a non-chiral way, e.g. as $\bar{\Phi}_f^{\text{op}}\Phi_f^{\text{op}}$. In principle this does not yet imply that the effective action cannot be described by the geometry of a chiral superfield: If all non-chiral contributions vanish after taking the vacuum expectation value, the effective action depends on $\Phi = \bar{D}^2\Phi_f$ only and the chiral geometry is restored with all the problems discussed in this work. If the non-chiral expressions do not vanish on the level of classical fields all descriptions based on the chiral extension are essentially incorrect. This especially means that the true ground state of SYM theories can only be found by considering gauge-symmetry breaking sources and the associated operators must have an important influence on the structure of the ground state! Besides the fact that supergauge symmetry now breaks dynamically, probably no straightforward interpretation of such a result could be given as the low energy structure of SYM now could depend on the unphysical components of the prepotential.

Within the context of non-standard geometries we should address another problem: the representation of the glueball. Indeed we have tacitly assumed that the operator F^2 is directly related to the lowest glueball state. This is not at all obvious but emerges from the fact that F^2 is the only purely gluonic, renormalizable and gauge-invariant operator. In principle there exists a simple way to keep L consistently an auxiliary field: we have to introduce different operators for the second-generation auxiliary field and the glueball operators, respectively. Based on the Lagrangian by Veneziano and Yankielowicz this has been proposed by Farrar et al. [92]. The authors observe that after splitting the classical Lagrangian into two independent fields for the real and imaginary part respectively, the fields can be recombined to a new superfield U called a constrained three-form multiplet (introduced in [93] and used to describe a gauge theory based on a super three-form). This new superfield contains additional degrees of freedom identified with the glueballs, the original superfield is found again by the relation $\bar{D}^2U \sim \Phi$ whereby U is real. The authors then construct an effective Lagrangian in U that cannot be written in terms of Φ and derive the spectrum of the glueball states from it. From our point

of view the procedure is problematic. The classical fields are not primary objects but rely on a source extension of the Lagrangian. The source extension corresponding to this extended system is not given in [92]; we suspect that it does not exist at all. From the relation $\bar{D}^2U \sim \Phi$ we conclude

$$\begin{aligned} U^{\text{op}} &\sim A^\alpha W_\alpha + X \\ \bar{D}^2 X &= 0 \Rightarrow X = \bar{D}^\alpha Y_\alpha, \end{aligned} \quad (26)$$

with $A_\alpha = -i(e^V D_\alpha e^{-V})$ representing the spinorial connection. It is easy to check that the reality condition is incompatible with the above structure. If the source extension shall be defined on the basis of the operator superfield $U^{\text{op}} \sim A^\alpha W_\alpha + \text{h.c.}$ all the terms not compatible with the constrained three-form multiplet would have to vanish when taking the vacuum expectation value. But with this choice we immediately get into new difficulties: Obviously the so-called glueball states (e.g. the lowest component of U , identified with the scalar glueball) are not supergauge invariant and are moreover not at all gluonic operators but depend on the non-gluonic physical fields as well as on the non-physical ones. The example shows that it is very difficult to include glueball states which are not associated with F^2 . Of course this does not mean that our solution must be correct, but within all approaches discussed in this work there is simply no way to describe a glueball not associated with the classical field of F^2 . A different picture could be described within a model including non-renormalizable operators as low energy degrees of freedom only.

Recently it has been speculated that the operator $A_\mu A^\mu$ could have a non-trivial meaning in the QCD vacuum [94, 95]. The local version of this operator is (due to gauge invariance) no candidate for a glueball state, but the integral over it could indeed be relevant. We are not able to give further comments on these calculations at this point. Even if this would be a promising ansatz to understand non-perturbative effects in the gluonic sector it would probably be very difficult to include this into our approach to SYM.

7 Some comments on more complicated models

Let us finally make some remarks on more complicated models: Can the inclusion of matter help? We are not able to give a final answer but would like to point out some problems: If we consider SQCD with large masses the only new contribution to the trace anomaly (the quark condensate) has again the wrong sign from the point of view of supersymmetry. If the masses are small the situation is more complicated due to possible contributions from scalar VEVs. At zero mass the new problem of a classical moduli space arises. In the LEEA approximation [39–41] the latter is found to be lifted and the low energy structure is again a SYM theory. In the light of our discussions this result is again problematic.

We want to analyze $N = 2$ SYM a little bit more in detail. Within $N = 2$ superspace the action is given by

$$\begin{aligned} \mathcal{L} &= \int d^4\theta \tau \text{Tr}(W^2) + \text{h.c.} \\ &= \frac{1}{C(G)} \text{Tr} \left(\frac{1}{g^2} [D_\mu, \bar{C}] [D^\mu, C] + \frac{i}{g^2} \lambda^{i\alpha} \sigma_{\alpha\dot{\alpha}}^\mu [D_\mu, \bar{\lambda}_i^{\dot{\alpha}}] \right. \\ &\quad - \frac{1}{4g^2} F_{\mu\nu} F^{\mu\nu} - \frac{\vartheta}{32\pi^2} F_{\mu\nu} \tilde{F}^{\mu\nu} \\ &\quad + \frac{1}{4g^2} H_{\{ij\}} H^{\{ij\}} + \frac{1}{g^2} C [C, \bar{C}] \bar{C} + \frac{i}{\sqrt{2}g^2} C \{ \bar{\lambda}_i^{\dot{\alpha}}, \lambda_i^{\dot{\alpha}} \} \\ &\quad \left. - \frac{i}{\sqrt{2}g^2} \bar{C} \{ \lambda_i^\alpha, \lambda_i^\alpha \} \right), \end{aligned} \quad (27)$$

with the chiral $N = 2$ multiplet (written in $N = 2$ Wess-Zumino gauge)

$$\begin{aligned} W(x, \theta_i^\alpha) &= \sqrt{2}C(x) + \sqrt{2}\theta_i^\alpha \lambda_i^{\dot{\alpha}}(x) + \theta^{\alpha\beta} v_{\alpha\beta}(x), \\ &\quad + \theta_{ij} H^{ij}(x) + \vartheta_i^\alpha \chi_\alpha^i(x) + \theta^4 D(x), \\ v_{\alpha\beta} &= \frac{i}{2} \sigma_{\alpha\beta}^{\mu\nu} F_{\mu\nu}, \\ \chi_i^\alpha &= i\sqrt{2}(\bar{\sigma}^\mu)^{\dot{\alpha}\alpha} [D_\mu, \bar{\lambda}_i^{\dot{\alpha}}] + \frac{i}{\sqrt{2}} [\bar{C}, \lambda_i^\alpha], \\ D &= \sqrt{2} [D^\mu, [D_\mu, \bar{C}]] - \frac{1}{\sqrt{2}} [\bar{C}, [C, C]] \\ &\quad - i \{ \bar{\lambda}_i^{\dot{\alpha}}, \lambda_i^{\dot{\alpha}} \}. \end{aligned} \quad (28)$$

When breaking $N = 2$ SYM we have two choices: We can break directly both supersymmetries or we can break the theory down to $N = 1$. Let us look at the first case. We define the source multiplet

$$\begin{aligned} J(x) &= \tau(x) + \theta_i^\alpha \zeta_\alpha^i - 4\theta_{ij} m^{ij}(x) + \theta^{\alpha\beta} w_{\alpha\beta} \\ &\quad + \vartheta_i^\alpha \kappa_\alpha^i + 4\theta^4 M^2(x), \end{aligned} \quad (29)$$

which has to obey the stability condition [8]

$$\text{Re}(\tau) \geq 0, \quad g^2 \rho^2 \geq |M^2 + g^2 \mu^2|, \quad (30)$$

with $\mu^2 = \tilde{m}_A \tilde{m}_A$, $\rho^2 = \tilde{m}_A \tilde{m}_A$; $m^{ij} = -i((\tau_A)^i_j / 2) \tilde{m}_A$.

Note that the stability constraint forbids one to choose the scalar mass term M^2 to be non-zero while keeping the fermions massless ($m^{ij} = 0$). To establish the non-decoupling theorem of $N = 2 \rightarrow N = 0$ we should start from the trace anomaly of $N = 2$: $T^\mu{}_\mu = -(\beta/g)\mathcal{L}$. In contrast to $N = 1$ the Lagrangian is now much more complicated and we cannot assume that the field-strength tensor is the only physical operator receiving a vacuum expectation value. Moreover the energy-momentum tensor is conserved for vanishing sources only, and thus we cannot expect that the trace anomaly is $\sim \mathcal{L}$ for $J \neq 0$. We thus have to define a covariant energy-momentum tensor that is conserved for any value of the sources [74]. The basic structure of the trace anomaly of this covariant e.-m. tensor for any value of the sources must be given by

$$T^\mu{}_\mu = -\frac{\beta}{g} A + \frac{2}{C(G)} (M^2 \text{Tr} C^2 + \text{h.c.}) -$$

$$\begin{aligned} &- \frac{1}{C(G)} (m_{ij} \text{Tr} \lambda^i \lambda^j + \text{h.c.}) \\ &+ \frac{4}{C(G)} \text{Tr} (m_{ij} C + \bar{m}_{ij} \bar{C})^2 + B, \end{aligned} \quad (31)$$

where $A = -1/(4C(G))F_{\mu\nu}F^{\mu\nu} + A'$, and B represents any additional terms independent of the sources. Now we split the β -function into

$$\beta(g) = -b_{\text{YM}} + b_\lambda + b_C. \quad (32)$$

Without dropping any terms the non-decoupling theorem then reads (omitting the ground-state brackets)

$$\begin{aligned} &- \frac{b_\lambda + b_C}{g} A + \frac{b_{\text{YM}}}{g} A' \\ &= - \lim_{m \rightarrow \infty} \left(\frac{2}{C(G)} (M^2 \text{Tr} C^2 + \text{h.c.}) \right. \\ &\quad - \frac{1}{C(G)} (m_{ij} \text{Tr} \lambda^i \lambda^j + \text{h.c.}) \\ &\quad \left. + \frac{4}{C(G)} \text{Tr} (m_{ij} C + \bar{m}_{ij} \bar{C})^2 + B \right). \end{aligned} \quad (33)$$

Here $m \rightarrow \infty$ means any limit degenerating the theory to YM and respecting the stability constraints. If the fermions and scalars decouple from each other in the heavy mass limit (i.e. the VEVs of fermionic operators do not depend on the scalars and vice versa) we recover the standard non-decoupling theorem if $\langle \Omega | A' | \Omega \rangle = \langle \Omega | B | \Omega \rangle = 0$. Using a formulation of the sources in terms of $N = 1$ superfields one can see that the non-decoupling theorem of the fermions agrees with the one of a single Dirac fermion. A note about the auxiliary fields: Of course they could have a VEV for vanishing sources, but as discussed for $N = 1$ we expect the latter to disappear at some finite scale of the mass parameters.

More interesting than breaking both supersymmetries turns out to be the partial supersymmetry breaking. Now only one component of m^{ij} is growing large, giving a mass to the scalars as well as to one gluino. The trace anomaly need not be $N = 2$ supersymmetric, but it must still respect $N = 1$ supersymmetry. We then end up with the following non-decoupling theorem for the fermions:

$$\begin{aligned} &\frac{b_\psi}{g^3 C(G)} \langle \Omega | \left(\frac{1}{4} \text{Tr} F_{\mu\nu} F^{\mu\nu} - \frac{1}{2} \text{Tr} D^2 \right) | \Omega \rangle \\ &= - \lim_{m \rightarrow \infty} \frac{m}{2C(G)} \langle \Omega | \text{Tr} \psi \psi | \Omega \rangle, \end{aligned} \quad (34)$$

where ψ is the massive gluino in the $N = 1$ language. We argued in this work that

$$\langle \Omega | ((1/4) \text{Tr} F_{\mu\nu} F^{\mu\nu} - (1/2) \text{Tr} D^2) | \Omega \rangle < 0$$

which would imply $\langle \Omega | \text{Tr} \psi \psi | \Omega \rangle > 0$, a clearly unacceptable result. To banish again the vacuum expectation value of the auxiliary field does not really cure the problem. It would imply $\langle \Omega | \text{Tr} \psi \psi | \Omega \rangle \rightarrow 0$ for $m \rightarrow \infty$ which does not at all fit with our expectations. This short analysis

shows that our arguments about $N = 2$ have probably been too simple; similar questions may arise when studying the signs of the scalar condensates. We cannot yet give an acceptable interpretation of this problem but have to postpone this to future investigations. Of course SQCD will have a similar behavior when decoupling some quark superfields.

Let us finally look at $N = 2$ SYM near the symmetry conserving point. Is there need for a vacuum expectation value of the auxiliary fields as in $N = 1$? When reducing the potential to

$$V = V(\phi) = \frac{1}{g^2 C(G)} \text{Tr}[\phi, \phi^\dagger]^2 \quad (35)$$

as done by Seiberg and Witten [37] all contributions to the trace anomaly are negative semi-definite

$$\begin{aligned} \langle \Omega | T^\mu{}_\mu | \Omega \rangle &= -\frac{\beta}{g} \langle \Omega | \mathcal{L} | \Omega \rangle \\ &\rightarrow \frac{\beta}{g^3 C(G)} \langle \Omega | \text{Tr}[\phi, \phi^\dagger]^2 | \Omega \rangle \leq 0, \end{aligned} \quad (36)$$

and supersymmetry is broken if and only if the auxiliary field changes its character. Completely analogous to $N = 1$ the problem of a phase transition emerges. The situation is even more delicate than in $N = 1$: If the Seiberg–Witten solution is correct while $N = 1$ is broken, $N = 2$ needs not only be protected from YM theory but also from $N = 1$ SYM and we end up with a phase transition in the gluino mass. This phase transition is again expected at $m = 0$ and is associated with a jump in the modulus. If on the other hand the $N = 2$ auxiliary fields get a vacuum expectation value, the auxiliary field of the matter field must be non-trivial at least for small values of the breaking parameters. This raises the question of the role of these auxiliary fields that could not be considered in this work. However, the structure of $N = 2$ SYM is much more complicated and we thus should be careful with these statements. It only shows that the solution by Seiberg and Witten suffers from the same problem as $N = 1$ SYM.

Considering the special structure of $N = 2$ theories a new question arises: It has been shown by Olive and Witten [96] that a theory with classically vanishing central charges can develop these dynamically leading to magnetic monopoles. This behavior is an important assumption in the solution by Seiberg and Witten for $N = 2$ SYM and SQCD. The existence of magnetic monopoles within a full quantum theory (i.e. not as semiclassical approximation) is however unclear. Striebel [97] showed that the finite energy solutions of the magnetic monopoles solely exist if the background field of the full gluon sector vanishes. This does not have direct implications for the solution by Seiberg and Witten. These authors look at the gauge group $SU(2)$ as broken down to $U(1)$ only. Then we indeed expect that F^2 vanishes. When choosing a more complicated gauge group however we will in general end up with partial breaking of gauge symmetry. $F^2 \neq 0$ then implies the absence of magnetic monopoles and the dynamical generation of central charges is endan-

gered. Within theories with more than one supersymmetry this scenario could possibly lead to a second important constraint (besides the discussed sign of F^2).

8 Summary and conclusions

In this paper we presented the basic tools needed to test dynamical symmetry breaking in the context of supersymmetric quantum field theories and we discussed the application thereof to the simplest interesting model, $N = 1$ SYM.

We showed that $N = 1$ SYM comprises a unique source extension, being covariant under all supersymmetries while conserving gauge symmetry. From the fundamental concept of studying symmetry breaking as a hysteresis effect, this source extension alone can and must be used to answer the question of dynamical supersymmetry breaking. We explained in detail the connection of our procedure to the thermodynamical limits and classified it in the context of exactly solvable as well as other field theories: In a four-dimensional theory with a non-perturbative sector (or any other theory that cannot be solved axiomatically) this concept is the only one which allows us to *determine* the ground state. We discussed in detail why other low energy approximations (some of these also called effective action or effective Lagrangian but conceptually different from the quantum effective action) are not suitable tools to answer the question of dynamical supersymmetry breaking – nevertheless, after identifying the correct ground state from a complete study of thermodynamical limits (or after extracting it from experimental results) these concepts can be useful to describe the dynamics over this state.

We discussed the relevant thermodynamical limit explicitly under the assumption that extrinsic supersymmetry is realized on the effective potential as superspace geometry on the level of the classical fields. We discussed in detail the assumption that can and must be made in order to be able to make any statements but leave the question of dynamical supersymmetry breaking open. We gave some comments about this construction from the point of view of perturbation theory. Especially we motivated that unbroken supersymmetry in the non-perturbative region is rather assumed than found as a result.

Comparing the semiclassical analysis of QCD with our result leads to the observation that the sign of the vacuum energy (being defined as the vacuum expectation value of the energy-momentum tensor) of SYM lies in a unphysical region from the point of view of QCD. This raises the question of possible connections of these two theories. On the semiclassical level this can be established by means of non-decoupling theorems and they lead to a surprising conclusion: The specific form of the superpotential resulting from supersymmetric non-linear σ -models implies the existence of a phase transition separating supersymmetric theories from non-supersymmetric ones. We discussed this in detail for $N = 1$ SYM and found that the phase transition in the gluino mass must be at $m = 0$. However, a supersymmetric theory protected by a phase tran-

sition would not obey minimal thermodynamical stability conditions and the infrared problem at the supersymmetry preserving point is not getting removed. Moreover a phase transition leads to a serious conceptual problem: Lacking understanding of non-perturbative effects, they are included in all standard approximations by assumption in comparison with theories realized in nature. Such assumptions based on physical arguments are clearly unfounded if the theory does not have any connection to a physically relevant model.

To get an acceptable behavior we conclude that the phase transition does not exist. The effective potential receives important contributions that cannot be written in terms of standard non-linear σ -models. This is closely related to the question of the relevance of the auxiliary fields, as the potential of the latter is now getting changed by non-perturbative effects. This removes all potential instabilities in the infrared region, but supersymmetry is completely run over by chiral symmetry breaking, confinement and the dynamical formation of a mass gap. We are not able to show from first principles that supersymmetry must break dynamically, but even assuming the possibility of conserved supersymmetry in this new scenario has drastic consequences. The low energy behavior thereof would be completely different than the standard solutions from effective Lagrangians or instanton calculations: All these results are based on the assumption that supersymmetry on the level of classical fields is still realized as an integral over superspace. For our new solution we have to modify this picture.

At the current status of the discussion we are not able to decide whether $N = 1$ SYM breaks supersymmetry dynamically or not. However the illustration of the different arguments and restrictions in favor of and against dynamical supersymmetry breaking unraveled the following point: Within the theoretical discussion there exists an important difference between theories accessible in experiments (QCD) and others (SYM or SQCD). For the first class different approximations to the low energy dynamics using semiclassical and/or momentum expansion have been very successful. The application of such techniques often draw upon the known vacuum structure of the theory and thus describe the dynamics over this structure only. In supersymmetry this restriction is obviously impossible and thus it has been tried to use the same techniques for both, the determination of the vacuum structure and the description of the dynamics. This ansatz led to a consistent description of many supersymmetric gauge theories.

Our discussion shows that these results are nevertheless problematic. All tools used above factor out important aspects of the low energy dynamics, namely non-perturbative non-semiclassical effects. Though the consistency of the standard picture of supersymmetry strongly suggests a coherence of all these models within the given approximation, we showed that important non-semiclassical effects cannot be excluded, neither by LEEAs, nor by instanton calculations or by the Witten index. We thus propose to re-analyze the structure of the ground state including the

full dynamics of the system, which is done by using the QEA as fundamental object. Such an ansatz asks for a completely new interpretation of superspace geometry and of the role of the auxiliary fields. Many aspects therein are still unclear and though some promising progress towards an understanding of such models has been made recently (presented in [9]), a concrete model describing at least $N = 1$ SYM is not yet in sight. Whatever such a model will look like, we can foresee that it will not be compatible with the LEEA or effective Lagrangian description of the theory, as the standard superspace geometry used in the latter approaches cannot be relevant in the first case. In this situation we conclude in analogy to QCD that the QEA must be the more fundamental object. Therefore in our opinion the LEEA has to conform to the results from the QEA.

Many other questions are still open. Even on the level of $N = 1$ SYM the vacuum expectation value of the Lagrangian now consists of two different operators that cannot be separated in a supersymmetry covariant way. However, independent knowledge about each of them is needed to learn about the principles of supersymmetry breaking, especially about the goldstino coupling. In more complicated models this becomes even more important. As an example confinement could be realized in $N = 2$ SYM by trilinear Yukawa-like condensates, but supersymmetry connects the corresponding operators to the scalar potential and to $F_{\mu\nu}F^{\mu\nu}$. When coupling matter fields to $N = 1$ or when breaking $N = 2$ explicitly to $N = 1$, the role of the matter auxiliary field F must be studied. The latter can no longer be eliminated naively: In $N = 2$ F and D are related by the internal $SU(2)$ symmetry and more generally the elimination leads to unacceptable decoupling behaviors. An interpretation could possibly be found by studying the structure of the energy-momentum tensor: Therein the F fields are important for the correct breaking of superconformal invariance and could play a similar role as the D field in $N = 1$ SYM.

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